Jan 22, 2014

Warm-up:

(1) $\frac{d}{dx}\left(\sqrt[3]{x}+\frac{3}{x}+5\right)=\frac{d}{dx}\left(x^{1/3}+3x^{-1}+5\right)$

$=\frac{1}{3}x^{-2/3}+3(-1)x^{-2}+5$

$=\frac{1}{3}x^{-2/3}-3x^{-2}+5$

(2) $\frac{d}{dx}\sqrt{\frac{x^2+1}{x^2+2}} = \frac{\frac{x}{\sqrt{x^2+1}}\cdot(x^2+2)-2x\cdot\sqrt{x^2+1}}{(x^2+2)^2}$ $\quad \frac{f'g-g'f}{g^2}$

$f(x)=\sqrt{x^2+1}$: $f(x)=h(k(x))$: $f'(x)=h'(k(x))\cdot k'(x)$

$h(x)=\sqrt{x}$ $\quad k(x)=x^2+1$ $\quad =\frac{1}{2\sqrt{x^2+1}}\cdot 2x = \frac{x}{\sqrt{x^2+1}}$

$g(x)=x^2+2$

$g'(x)=2x$

Trig Derivatives

Why? trig functions model periodic phenomona
- light + sound waves
- springs

- Derivative of $\sin x$:

$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$ $\quad$ Recall: $\sin(x+h)=\sin x\cos h+\sin h\cos x$

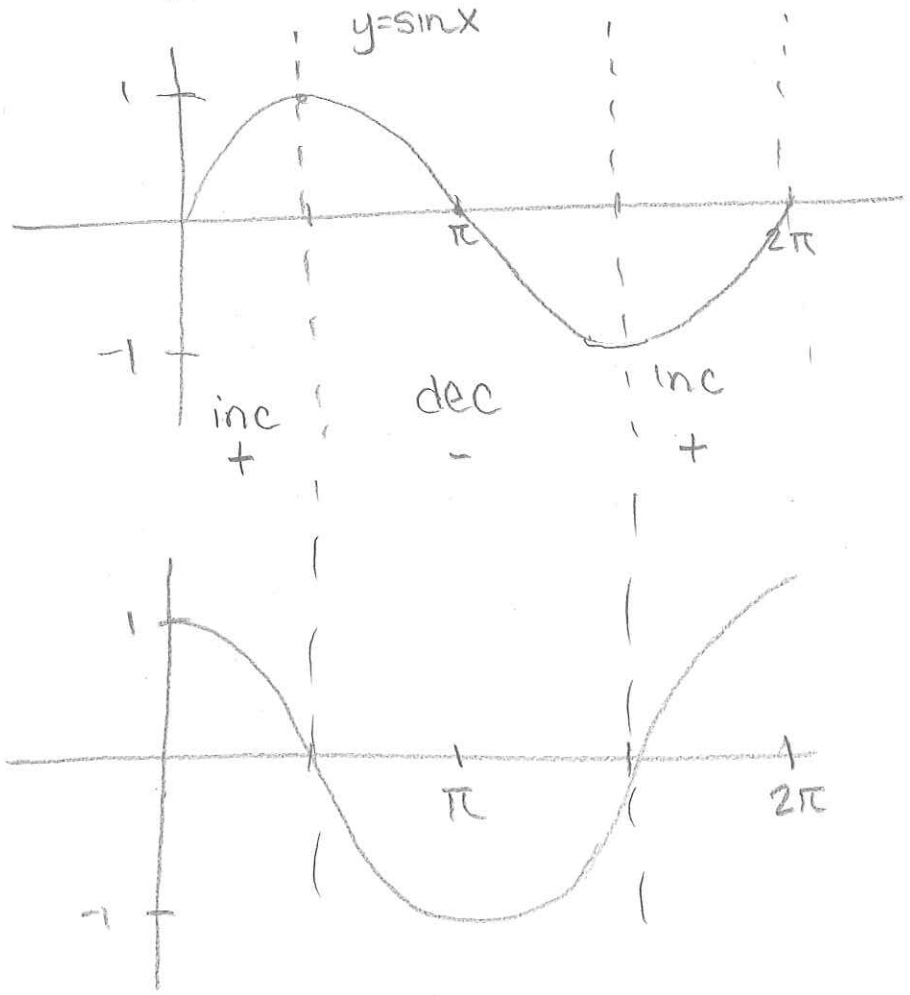
$=\lim_{h\to 0}\frac{\sin x\cos h+\sin h\cos x-\sin x}{h}$

$=\lim_{h\to 0}\frac{\sin x\cos h-\sin x}{h}+\lim_{h\to 0}\frac{\sin h\cos x}{h}$

$=\sin x\underbrace{\lim_{h\to 0}\frac{\cos h-1}{h}}_{0}+\cos x\underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{1}$

$=\cos x$

Graphically,



$$\frac{d}{dx}\cos x = \frac{d}{dx}\sin\left(\frac{\pi}{2}-x\right)$$

$$\overset{\text{(chain rule)}}{=} \cos\left(\frac{\pi}{2}-x\right)\cdot(-1)$$

$$= -\cos\left(\frac{\pi}{2}-x\right)$$

$$= -\sin x$$

FACT: $\sin\left(\frac{\pi}{2}-x\right) = \cos x$

$\cos\left(\frac{\pi}{2}-x\right) = \sin x$

THM: $\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

Ex: (1) $\frac{d}{dx} 4\sin^2 x$

Two ways: Product – $\frac{d}{dx} 4\sin x \cdot \sin x = 4\sin x \cdot \cos x + 4\cos x \sin x$

$= 8\sin x \cos x$

Chain – $\frac{d}{dx} 4(\sin x)^2 = 8(\sin x) \cdot \cos x$

(2) $\frac{d}{dx} \underbrace{2\sin(x^2+1)}_{\text{chain rules}} + 2x = 2\cos(x^2+1) \cdot 2x + 2$

$f'(g(x)) \cdot g'(x)$

$f(x) = 2\sin x$

$g(x) = x^2+1$

$f(g(x)) = 2\sin(x^2+1)$

$f'(x) = 2\cos x$

$g'(x) = 2x$

(3) $\frac{d}{dx} \underset{f(x)\cdot g(x)}{\sin x \cos x} = \underset{f}{\sin x} \underset{g'}{(-\sin x)} + \underset{f'}{\cos x} \underset{g}{\cos x}$

$= -\sin^2 x + \cos^2 x = \cos^2 x - \sin^2 x$

(4) $\frac{d}{dx} \cos^2(\sin x) = \frac{d}{dx} (\cos(\sin x))^2 = 2(\cos(\sin x)) \cdot (-\sin(\cos x)) \cdot \cos x$

$f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$

Chain Rule:

$f(x) = x^2$ $g(x) = \cos x$ $h(x) = \sin x$

$f(g(h(x))) = \cos^2(\sin x)$

$f'(x) = 2x$

$g'(x) = -\sin x$

$h'(x) = \cos x$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x}$$

$$= \frac{\cos x \cdot \cos x - (-\sin x)\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

Recall Trig:

$\tan x = \frac{\sin x}{\cos x}$

$\sec x = \frac{1}{\cos x}$

$\csc x = \frac{1}{\sin x}$

$\cot x = \frac{\cos x}{\sin x}$

$$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = \frac{d}{dx}(\cos x)^{-1} = -1(\cos x)^{-2} \cdot -\sin x$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \cdot \sec x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$